Astro 596/496 NPA

Lecture 17

February 25, 2019

Announcements:

- **Problem Set 3** out, due next Friday

  Office Hours Wed 2-3pm or by appointment

  if questions arise will post Discussion on Compass

Last time: began thermal history of the Universe

early U expansion radiation dominated: $H^2 \approx 8\pi G \rho_{\rm rad}/3$

Q: at any T, what determines $\rho_{\rm rad}$?

Q: when/why does g_* change? how should $g_*(T)$ evolve with T?

thermal radiation (energy) density:

for boson (fermion) species each with its T_{b} (T_f)

$$\rho_{\mathrm{rel}} = \sum_i \rho_{\mathrm{rel},i} = g_*(T)\ \frac{\pi^2}{30}\frac{k^4 T^4}{\hbar^3 c^3}$$

where $T = T_\gamma$ is photon temperature, and

where g_* counts **"effective relativistic degrees of freedom"**

$$g_*(T) = \sum_{\mathrm{bosons}} g_{\mathrm{b}} \left(\frac{T_{\mathrm{b}}}{T}\right)^4 + \sum_{\mathrm{fermions}} \frac{7}{8} g_{\mathrm{f}} \left(\frac{T_{\mathrm{f}}}{T}\right)^4$$

g_* *changes* when species become *non-relativistic*, i.e., $T < m_i$

thus *decreases* with time

consider epochs when $g_* \approx const$

 Q: $\rho_{\mathrm{rad}}(a)$*?* $a(t)$*?* $H(t)$*?* $T(t)$*?* $t(T)$*? effect of* g_**?*

Radiation Domination: Time and Temperature Evolution

when $g_* \approx const$: relativistic species $T_i \propto a^{-1}$:

$$\left(\frac{\dot{a}}{a}\right)^2 \sim \rho_{\rm rad} \sim g_* T^4 \sim \frac{g_*}{a^4} \tag{1}$$

$$a \ da \sim g_*^{1/2} \ dt \tag{2}$$

and integration gives

$$a \sim g_*^{1/4} \ t^{1/2} \tag{3}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{2t} \tag{4}$$

$$T \sim \frac{1}{a} \sim \frac{1}{g_*^{1/4} \ t^{1/2}} \tag{5}$$

$$t \sim \frac{1}{g_*^{1/2} \ T^2} \tag{6}$$

ω *higher* $g*$ means *faster expansion*

Q: g_* *at BBN onset* $T \gtrsim few$ MeV*?*

BBN Initial Conditions: Radiation Domination

total relativistic energy density at BBN onset:

$$\rho_{\rm rel} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_{1\nu\bar{\nu}} \equiv g_* \frac{\pi^2}{30} T^4 \qquad (7)$$

$$g_* = g_*(\gamma) + g_*(e^\pm) + \sum_{\text{neutrinos},i} g(\nu_i \bar{\nu}_i) \qquad (8)$$

pairs: at $T \gtrsim 1$ MeV, $e^\pm$ are relativisitc
for each, $s = 1/2$, so $g = 2$ spin degrees of freedom

$$g_*(e^\pm) = g_*(e^-) + g_*(e^+) = \frac{7}{8} \cdot 2 \cdot 2 = \frac{7}{2} \qquad (9)$$

neutrinos: for sure $m_\nu \lesssim 1\text{eV} \ll T$
also assume $\mu_\nu \ll T \rightarrow$ absolute n_ν, ρ_ν, P_ν set by T_ν; equivalent to $n_L \sim n_B$
each species $\nu_i \in (\nu_e, \nu_\mu, \nu_\tau)$ has $g(\nu_i) = 1$ helicity
neutrinos are born left handed only! "maximally parity violating"

$$g_*(\nu_i \bar{\nu}) = g_*(\nu_i) + g_*(\bar{\nu}_i) = \frac{7}{8} \cdot 2 = \frac{7}{4} \qquad (10)$$

BBN Initial Conditions: Expansion History and Rate

relativistic degrees of freedom at BBN start:

$$g_* = g_*(\gamma) + g_*(e^{\pm}) + \sum_{\text{neutrinos},i} g(\nu_i\bar{\nu}_i) = \frac{22 + 7N_\nu}{4} = \frac{43}{4}$$

Friedmann gives:

$$\boxed{\frac{t}{1\ \text{sec}} \approx \left(\frac{1\ \text{MeV}}{T}\right)^2} \qquad (11)$$

expansion rate

$$H = \frac{1}{2t} \approx 1\ \text{sec}^{-1}\ \left(\frac{T}{1\ \text{MeV}}\right)^2 \qquad (12)$$

now focus on baryons *Q: what sets* n_B*?* n/p*?*

BBN Initial Conditions: The Baryons

Cosmic baryon density n_B, and thus $\eta = n_B/n_\gamma$
not changed by reactions with $T \lesssim E_{\text{Fermilab}} \sim 1$ TeV $= 10^6$ MeV
i.e., baryon non-conservation not observed to date

▷ n_B set somehow in early universe ("cosmic baryogenesis")

▷ don't *a priori* know n_B, treat as free parameter (η)

neutron-to-proton ratio n/p *can* and *does* change at ~ 1 MeV

Q: what kind of interaction needed to change $n \leftrightarrow p$*?*

Q: what happens to n, p *if such reactions are "fast"?*

for $n \leftrightarrow p$ interchange

nucleon (quark) type must change

$\Rightarrow$ only happens in **weak** interactions

neutrino and electron interactions allow nucleon interconversion

$$n + \nu_e \leftrightarrow p + e^- \quad (13)$$

$$p + \bar{\nu}_e \leftrightarrow n + e^+ \quad (14)$$

when rates "fast":

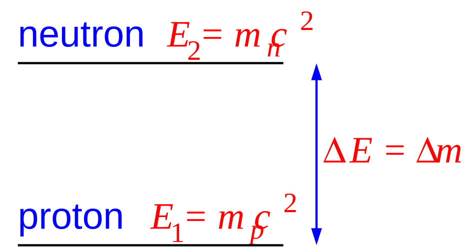
- *n, p system driven to ("chemical") equilibrium*
  zero net reaction rate per volume: $r(n \to p) = r(p \to n)$
- $(n/p) \to (n/p)_{\rm eq}$

Note: since weak interactions fast, EM rxns also fast:

$\Rightarrow$ all particles thermal, w/ same T

while weak interaction is fast, i.e., *in equilibrium*
n/p ratio is “thermal”

think of as *2-state* system: the “nucleon,”



- nucleon “ground state” is the *proton*
- nucleon “excited state” is the *neutron*

when in equilibrium Boltzmann gives state ratio:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2-E_1)/T} = e^{-(m_n-m_n)/T} \tag{15}$$

Q: behavior at low T? high T? what sets these regimes?

Neutron-to-Proton Evolution: Equilibrium

equilibrium neutron-to-proton ratio

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \tag{16}$$

scale set by nucleon mass difference

$$\Delta m = m_n - m_p = 1.293318 \pm 0.000009 \text{ MeV} \tag{17}$$

at $T \gg \Delta m$: $n/p \simeq 1$

at $T \ll \Delta m$: $n/p \simeq 0$

Equilibrium is Not Forever

when in equilibrium, U completely described by

- *temperature* T and
- *conserved quantum numbers* (via chem potentials μ)

Universe would be boring if always in equilibrium!

Happily, fell out of eq. now and then: "freeze-out"

freezeouts are most interesting times in cosmology

BBN, CMB, dark matter, baryon excess all stem from freezeouts

$n \leftrightarrow p$ *equilibrium* only holds

while weak reactions can maintain it

Q: What would cause equilibrium to fail?

Q: How would you quantify when eq fails?

Cosmic Freezeouts

useful rule: a reaction is

(1) in equilibrium when

conversion rate per nucleon $\Gamma \gg H$ *Hubble rate*

i.e., mean lifetime $\ll$ expansion time

or, mean free path $\ll$ horizon size $\sim ct \sim cH^{-1}$

(2) "frozen out" when $\Gamma \ll H$

Suggests "freezeout" criterion

$$\Gamma \overset{\text{freeze}}{=} H$$

so T_{f} set by: $H(T_{\text{f}}) = \Gamma(T_{\text{f}})$

Weak $n \leftrightarrow p$ Rates

example: want rate $\Gamma_{\text{per}\,n}$ of $\nu + n \rightarrow e^- + p$
as function of T

Generally,

$$\Gamma_{\text{per}\,n} = n_\nu \langle \sigma v \rangle \sim T^3 \langle \sigma \rangle \tag{18}$$

since $v_\nu \simeq c$

can show: cross section $\boxed{\sigma \sim \sigma_0 (E_\nu / m_e)^2 \sim E_\nu^2}$
where $\sigma_0 \sim 10^{-44}\ \text{cm}^2 \sim 10^{-19} \sigma_{\text{nuclear}}$ very small!
so *thermal average:* $\langle \sigma \rangle \sim \sigma_0 (T/m_e)^2$

for experts: $\sigma \sim G_F^2 T^2 \sim \alpha_{\text{weak}} T^2 / M_W^4$
so $\Gamma_{\text{weak}} \sim \alpha_{\text{weak}} T^5 / M_W^4$

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\text{weak}}$, i.e.,

$$\sqrt{G_{\text{grav}}}T^2 \sim \sigma_0 m_e^{-2} T^5 \tag{19}$$

$$\Rightarrow T_{\text{weak freeze}} \sim \frac{(G_{\text{grav}})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim \text{1 MeV} \tag{20}$$

gravity & weak interactions conspire to give $T_{\text{f}} \sim m_e \sim B_{\text{nuke}}$!

for experts: note that $G_{\text{grav}} = 1/M^2_{\text{Planck}}$, so

$$\frac{T^2}{M_{\text{Pl}}} \sim \alpha_{\text{weak}} \frac{T^5}{M_W^2} \tag{21}$$

$$\Rightarrow T_{\text{freeze}} \sim \left(\frac{M_W}{M_{\text{Pl}}}\right)^{1/3} M_W \sim 1 \text{ MeV} \tag{22}$$

freeze at nuclear scale, but by accident!

Q: what happens to n, p then? what else is going on?

Interlude: Pair Annihilation

right after weak freezeout, T_γ drops below $m_e = 0.511$ MeV
pairs become nonrelativistic, annihilate: $e^+e^- \to \gamma\gamma$
- mass energy $\to$ back to radiation
- small leftover amount of e^-

⋆ a sort of "heating" but really just restores relativistic energy
T_γ never rises, but cooling is briefly slowed

⋆ since νs decoupled, don't receive pair energy
cooler than photons thereafter
can show: $T_\nu = (4/11)^{1/3} T_\gamma = 0.714 T_\gamma$
today, the (relativistic) cosmic neutrino backgrounds have
$T_{\nu,0} = 0.714 T_{\gamma,0} = 1.95$ K

⋆ if you can think of how to detect this *cosmic ν background*
let me know and we'll publish–you can even be second author!

The Short but Interesting Life of a Neutron

(1) at $T > T_f$, $t \sim 1$ s
$n \leftrightarrow p$ rapid
maintain $n/p = e^{-\Delta m/T}$

(2) at $T = T_f$,
fix $n/p = e^{-\Delta m/T_f} \simeq 1/6$
so n "mass fraction" is

$$X_n = \frac{\rho_n}{\rho_B} = \frac{m_n n}{m_n n + m_p p} \approx \frac{n}{n+p} \approx 1/7 \quad (23)$$

(3) until nuclei form,
free n decay: $\dot{n} = -n/\tau_n$, with $\tau_n = 885.7 \pm 0.8$ s
then mass fraction drops as

$$X_n = X_{n,i} e^{-\Delta t/\tau} \quad (24)$$

Q: why take this simple from?

Deuterium Bottleneck

Build complex nuclei from n, p

first step: *deuterium production* $n + p \rightarrow d + \gamma$

`www: BBN reaction network`

energy release $Q = B(d) = E_\gamma = 2.22$ MeV: exothermic

reverse "photodissociation" $d + \gamma \rightarrow n + p$ allowed but *endo*thermic

Naïvely: at $T < T_f < Q$, too cold to photo-dis

But: $n_\gamma / n_B = 1/\eta \sim 10^9 \gg 1$

$\Rightarrow$ many photons per baryon

$\Rightarrow \langle E_\gamma \rangle < Q$, but many photons have $E_\gamma > Q$

D can't survive until $T \ll Q$!

c.f. delay in recombination–same idea

Q: How low to go?

Nuclear Statistical Equilibrium

For a non-relativistic species (Maxwell-Boltzmann):

$$n = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-(m-\mu)/T} \quad (25)$$

For $n(p,\gamma)d$ in *chemical equilibrium:* $\mu_n + \mu_p = \mu_d + \mu_\gamma = \mu_d$, (since $\mu_\gamma = 0$), so

$$\frac{n_n n_p}{n_d} = \left(\frac{(m_n m_p/m_d)kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_n+m_p-m_d)/T}$$

$$= \left(\frac{m_u kT/2}{2\pi\hbar^2}\right)^{3/2} e^{-B_D/T} \quad (26)$$

example of "nuclear statistical equilibrium" this example: Saha equation

write baryon fraction $Y_i = n_i/n_B$ and $n_B = \eta n_\gamma$

$$Y_d \sim Y_n Y_p \eta (T/m_u)^{3/2} e^{B_D/T} \quad (27)$$

Q: what is low-T behavior?

When $Y_d \to 1$: Nuke buildup starts

$$\ln Y_d \simeq B_D/T + \ln \eta + 3/2 \ln T/m_u \sim 0 \tag{28}$$

so

$$T_D \simeq \frac{B_D}{\ln \eta^{-1}} \sim 0.07 \text{ MeV} \tag{29}$$

i.e., nuke rxns begin at $T \simeq 10^9$ K Note: $T_D \ll B_2$ since $\eta \ll 1$

time $t_d \sim 200$ s $\rightarrow$ "the first 3 min"

between freezeout and T_D:
free n decay: mass fraction $X_n = X_{n,i} e^{-\Delta t/\tau} \simeq 0.12$

`www: nuke network` *Q: where is flow direction? why?*