Astro 596/496 NPA

Lecture 18

February 27, 2019

Announcements:

- **Problem Set 3** out, due next Friday

  Office Hours Wed 2-3pm or by appointment

  if questions arise will post Discussion on Compass

Last time: neutrons and protons in the early Universe

Q: when in equilibrium, what is n/p ratio?

Q: what reactions maintain this equilibrium?

Q: why does this equilibrium end? when?

Problem Set 3: Hints and Errata

- for numerical answers, $\eta = 6 \times 10^{-10}$ but you can use $\eta \sim 10^{-9}$ without penalty
- Question 2(b): *can skip last part* involving $\Omega_{\rm B} h^2$
- **Cosmic Entropy**

note that energy density is $\varepsilon = \varepsilon(T) = (\partial E/\partial_V)_T$
so that in 2nd law of thermo, $E = E(T, V) = \varepsilon(T)\, V$
similarly $S = s(T)\, V$, $N = n(T)\, V$

in notes, factor of 2 typo fixed:
Planck function for photon number density distribution

$$\frac{dn}{dE} = \frac{g}{2\pi^2 \hbar^3 c^2}\ E^2\ f(E) = \frac{g}{2\pi^2 \hbar^3 c^2}\ \frac{E^2}{e^{E/kT} - 1} \tag{1}$$

in early universe: $T \gg 1$ MeV:

neutrino and electron interactions allow nucleon interconversion

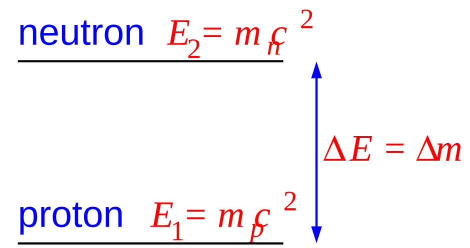
$$n + \nu_e \leftrightarrow p + e^- \quad (2)$$

$$p + \bar{\nu}_e \leftrightarrow n + e^+ \quad (3)$$

finds equilibrium: equal forward and reverse rates per volume

when in equilibrium Boltzmann gives:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2-E_1)/T} = e^{-(m_n-m_n)/T}$$



equilibrium requires *rate per nucleon* $\Gamma \gg H$ *Hubble rate*

lost at **freezeout**:

$$\Gamma \overset{\text{freeze}}{=} H$$

ω $n \leftrightarrow p$ freezeout: $T_{\text{freeze}} \approx 1\text{MeV}$, $t_{\text{freeze}} \approx 1$ sec

Interlude: Pair Annihilation

right after weak freezeout, T_γ drops below $m_e = 0.511$ MeV
pairs become nonrelativistic, annihilate: $e^+ e^- \to \gamma\gamma$

- mass energy $\to$ back to radiation
- small leftover amount of e^- to balance p charge

★ a sort of "heating" but really just restores relativistic energy
T_γ never rises, but cooling is briefly slowed

★ since νs decoupled, don't receive pair energy
cooler than photons thereafter
can show (**PS3**): $T_\nu = (4/11)^{1/3} T_\gamma = 0.714 T_\gamma$
today, the (relativistic) cosmic neutrino backgrounds have
$T_{\nu,0} = 0.714 T_{\gamma,0} = 1.95$ K

★ if you can think of how to detect this *cosmic ν background*
let me know and we'll publish–you can even be second author!

The Short but Interesting Life of a Neutron

(1) at $T > T_f$, $t \sim 1$ s
$n \leftrightarrow p$ rapid
maintain $n/p = (n/p)_{\rm eq} = e^{-\Delta m/T}$

(2) at $T = T_f$,
freezeout fixes $(n/p)_{\rm f} = e^{-\Delta m/T_f} \simeq 1/6$
so n "mass fraction" is

$$X_n = \frac{\rho_n}{\rho_B} = \frac{m_n n}{m_n n + m_p p} \approx \frac{n}{n+p} \approx 1/7 \qquad (4)$$

(3) until nuclei form,
free n decay: $\dot{n} = -n/\tau_n$, with $\tau_n = 885.7 \pm 0.8$ s
then mass fraction drops as

$$X_n = X_{n,i} e^{-\Delta t/\tau} \qquad (5)$$

Q: why take this simple from?

Deuterium Bottleneck

Build complex nuclei from n, p

first step: *deuterium production* $n + p \to d + \gamma$

`www: BBN reaction network`

energy release $Q = B(d) = E_\gamma = 2.22$ MeV: exothermic

reverse "photodissociation" $d + \gamma \to n + p$ allowed but *endo*thermic

Naïvely: at $T < T_f < Q$, too cold to photo-dissociate

But: $n_\gamma / n_{\rm B} = 1/\eta \sim 10^9 \gg 1$

$\Rightarrow$ many photons per baryon

$\Rightarrow \langle E_\gamma \rangle < Q$, but many photons have $E_\gamma > Q$

D can't survive until $T \ll Q$!

PS3: find when $n_\gamma(> Q) \sim n_{\rm B}$

Q: How low to go?

Nuclear Statistical Equilibrium

For a non-relativistic species (Maxwell-Boltzmann):

$$n = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-(m-\mu)/T} \tag{6}$$

For $n(p,\gamma)d$ in *chemical equilibrium:* $\mu_n + \mu_p = \mu_d + \mu_\gamma = \mu_d$, (since $\mu_\gamma = 0$), so

$$\begin{aligned} \frac{n_n n_p}{n_d} &= \left(\frac{(m_n m_p/m_d)kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_n+m_p-m_d)/T} \\ &= \left(\frac{m_u kT/2}{2\pi\hbar^2}\right)^{3/2} e^{-B_D/T} \end{aligned} \tag{7}$$

example of "nuclear statistical equilibrium" this example: Saha equation

write baryon fraction $Y_i = n_i/n_B$ and $n_B = \eta n_\gamma$

$$Y_d \sim Y_n Y_p \eta (T/m_u)^{3/2} e^{B_D/T} \tag{8}$$

Q: what is low-T behavior?

When $Y_d \rightarrow 1$: Nuke buildup starts

$$\ln Y_d \simeq B_D/T + \ln \eta + 3/2 \ln T/m_u \sim 0 \tag{9}$$

so (*big hint for PS 3!*)

$$T_D \simeq \frac{B_D}{\ln \eta^{-1}} \sim 0.07 \text{ MeV} \tag{10}$$

i.e., nuke rxns begin at $T \simeq 10^9$ K Note: $T_D \ll B_2$ since $\eta \ll 1$

time $t_d \sim 200$ s $\rightarrow$ "the first 3 min"

between freezeout and T_D:
free n decay: mass fraction $X_n = X_{n,i} e^{-\Delta t/\tau} \simeq 0.12$

∞ `www: nuke network` *Q: where is flow direction? why?*

Nuke reaction flow $\rightarrow$ highest binding energy $\rightarrow$ ^{4}He

almost all $n \rightarrow {}^4$He: $n({}^4\text{He})_{\text{after}} = 1/2\ n(n)_{\text{before}}$

$$Y_p = X({}^4\text{He}) \simeq 2(X_n)_{\text{before}} \simeq 0.24 \tag{11}$$

$\Rightarrow \sim 1/4$ of baryons into ^{4}He, 3/4 $p \rightarrow$H
result weakly (log) dependent on η
Robust prediction: large universal ^{4}He abundance

But nuke rxns also freeze out
$\Rightarrow n \rightarrow {}^4$He conversion incomplete
leftover traces of incomplete burning:

- D
- ^{3}He (and ^{3}H$\rightarrow^3$He)
- ^{7}Li (and ^{7}Be$\rightarrow^7$Li)

trace abundances $\leftrightarrow$ nuke freeze T
$\Rightarrow$ strong $n_B \propto \eta$ dependence

BBN theory: main result

- light element abundance predictions
- depend on baryon density $\leftrightarrow \eta \leftrightarrow \Omega_{\text{baryon}}$

"Schramm Plot' '

Lite Elt Abundances vs η

summarizes BBN theory predictions

`www: Schramm plot`

Note: no $A > 7$... *Q: why not?*

Why don't we go all the way to ^{56}Fe*?*

after all: most tightly bound

$\Rightarrow$ most favored thermodynamically (nuclear statistical equilibrium)

Why no elements $A > 7$?

1. *Coulomb barrier*
heavier products require heavier reactants
which have higher charges

2. nuclear physics: "mass gaps"
no stable nuclei have masses $A = 5, 8$
$\rightarrow$ with just p & ^{4}He, can't overcome via 2-body rxs
need 3-body rxns (e.g., $3\alpha \rightarrow {}^{12}$C) to jump gaps
but ρ, T too low

Stars *do* jump this gap, but only because have higher density a long time compared to BBN