Astro 596/496 NPA
Lecture 23
March 25, 2019

Announcements:

- **Problem Set 4 due Friday**

Last time, a long time ago:

- non-baryonic dark matter and physics
  Q: what's non-baryonic dark matter?
- Standard Model of particle physics
  fermionic matter, bosonic force carriers
  particle families *Q: how many? structure?*

Periodic Table of Elementary Particles

known fundamental particles: 3 families

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{array}{l} \text{charm quark} \\ \\ \\ \\ \end{array} \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{array}{l} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \\ \end{array} \tag{1}$$

+antiparticles

all spin-1/2: matter made of fermions!

for quarks and charged leptons, masses increase with each family
same for νs??

Particle Interactions: Who feels what?

all particles subject to gravity, and

neutrinos ν_e, ν_μ, ν_τ “feel” only weak interaction

charged leptons e, μ, τ feel only weak and EM

quarks feel all forces

Note: β decay really quark transformation:

$n \rightarrow p + e^- + \bar{\nu}_e$

$\boxed{udd} \rightarrow \boxed{uud} + e^- + \bar{\nu}_e$

so underlying reaction is

$d \rightarrow u + e^- + \bar{\nu}_e$

The Standard Model of Particle Physics: Twitter Version

- **3 matter families** of quarks and leptons

- massless neutrinos

- *fundamental interactions/forces: exchange of field quanta*
  structure set by internal *symmetry*
  - ▷ strong: quanta are gluons
  - ▷ electromagnetic: photon γ
  - ▷ weak: W^+,W^-,Z^0
  - ▷ and gravity (graviton?)

- **Higgs field:** scalar field, couples to all particles
  coupling strength → particle mass
  Higgs field quanta: spin-0 **Higgs boson**
  mass $m(H^0) = 125.18 \pm 0.16$ GeV largest known
  unstable: decays to everything! e.g., $H^0 \rightarrow b\bar{b}$

Non-Baryonic Dark Matter: Standard Model Candidates

Q: what Standard Model particles could *be non-baryonic dark matter?*

Q: hint–what Standard particles are stable?

Q: what is needed to tell if Standard Model particles are DM?

Standard Model Non-Baryonic Dark Matter

non-baryonic dark matter:
- *not baryons: quarks are out*
- *matter:* non-relativistic: photons are out
- *stable: Higgs,* $W^{\pm}$*,* Z^0 *out*

Leaves leptons

Charged leptons: e, μ, τ
only e stable, charge neutrality $n_e = n_Z$, $m_e \ll m_\mathrm{u}$
$\rightarrow \rho_e \ll \rho_B$

neutral leptons: neutrinos! ν_e, ν_μ, ν_τ
- not baryons!
- stable!
- weakly interacting
- families: get "three for price of one!"

excellent non-baryonic dark matter candidate!
...and the only Standard Model non-baryonic DM candidate!

Neutrino Dark Matter

neutrinos are a guaranteed component of non-baryonic dark matter!

even better: *we know the cosmic neutrino number density!*
recall: neutrinos freeze out at BBN, with $T_\nu = (4/11)^{1/3} T_\gamma$
so per $\nu\bar{\nu}$ neutrino species: $n(\nu\bar{\nu}) = (4/11) n_\gamma$
but: what is ρ_ν, Ω_ν? depends on neutrino masses

★ Laboratory studies of β decay
e.g., precision measurement of e^- energy in $^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$
place limits on ν_e mass *Q: how?*
current PDG limit: $\boxed{m(\nu_e) < 2 \text{ eV}}$

★ We shall see: *solar* and *atmospheric* νs
will ultimately show *all* 3 species have $\boxed{m(\nu) \lesssim few \text{ eV}}$

But you will show (PS 4): neutrino density parameter is

$$\Omega_\nu \simeq \frac{\sum_i m(\nu_i)}{45 \text{ eV}} \tag{2}$$

Q: implications for dark matter?

Q: implications for particle physics?

Dark Matter Requires New Physics

no viable particle dark matter candidates
in Standard Model of particle physics

non-baryonic DM **demands** *physics beyond the Standard Model*

Hugely important and exciting for particle physics!

Unlike dark energy: particle physics *does* offer solutions!
particle candidates available “off the shelf”
invented for particle physics motivations independent from DM!

- lightest supersymmetric particle
- axion
- strangelets...

(Almost) all of these are formed as *cold relics*

The Early Universe: Particle Dark Matter

Particle Dark Matter: Thermal Relics

Kolb & Turner, Ch. 5; Dodelson Ch. 3.4

if nonbaryonic stable particles created in Early U.
$\longrightarrow$ dark matter today (?)

consider an exotic particle ψ which is:

- *stable* (doesn't) decay
- created in early U., along with $\bar{\psi}$
- can *annihilate* via $\psi\bar{\psi} \leftrightarrow X\bar{X}$
  where X is some other, usually Standard Model, particle
  note: could be that $\psi = \bar{\psi} \rightarrow$ annihilations unavoidable!

if $\psi\bar{\psi}$ made in pairs (or $\psi = \bar{\psi}$)
$\longrightarrow$ thermally produce $n_\psi = n_{\bar{\psi}}$ in early U.
if only *annihilate* afterwards: $n_\psi = n_{\bar{\psi}}$ *always*
Q: if these are DM today, why don't they just annihilate?

Why Doesn't Dark Matter Annihilate?

If dark matter has $n_\psi = n_{\bar{\psi}}$
then complete annihilation $\psi\bar{\psi} \rightarrow X\bar{X}$
would indeed lead to $n_\psi \rightarrow 0$ today: boring!!

but *annhilations occur in expanding universe*
DM density and velocity is dropping!
eventually: *DM particles "don't find each other"*
freezeout of annihilations

so: annihilations don't ever stop
but just become rare → ψ becomes effectively stable
surviving particles: **relic abundance**

Q: what determines annihilation freezeout epoch?
Q: what leads to a relic abundance that's large? small?

Freezeout of Annihilations

we have already seen: *freezout* epoch (temperature $T_{\rm f}$) set by *reaction rate balance with expansion rate:*

$$\Gamma(T_{\rm f}) = H(T_{\rm f}) \tag{3}$$

annihilation rate per DM particle

$$\Gamma_{\rm ann} = n_\psi \langle \sigma_{\rm ann} v \rangle \tag{4}$$

- ▷ *before freezeout:* DM in equilibrium $n_\psi = n_{\psi,{\rm eq}}(T)$
- ▷ *after freezeout:* DM set by $n_{\psi,{\rm eq}}(T_{\rm f})$

freezout condition $H = n_{\psi,{\rm eq}} \langle \sigma_{\rm ann} v \rangle$
controlled by *annihilation cross section* via $\langle \sigma_{\rm ann} v \rangle$

consider DM non-relativistic at freezeout (“cold relic”)
Q: what is $n_{\psi,{\rm eq}}(T)$*?*

Cold Relics: WIMPs

cold relic: non-relativistic at freezeout
so with no chemical potential (number not conserved)

$$n_{\text{eq}}(T) \sim e^{-m/T}(mT)^{3/2} \tag{5}$$

Freezeout:
$\Gamma_{\text{ann}} = H$ at $T = T_{\text{f}}$
$\Rightarrow n_{\text{eq}} \langle \sigma v \rangle_{\text{ann}} \sim \sqrt{G} T^2$

Q: what needed to find value of T_{f}?

To solve:

- need annihilation cross section

  for many models, $\langle \sigma v \rangle \propto v^n$ (S-wave: $n = 0$)

  $\Rightarrow \langle \sigma v \rangle(T) = \sigma_1 c\ (T/m)^{n/2}$, where $\sigma_1 = \sigma_{E=m}$
- convenient rewrite $1/\sqrt{G} = M_{\mathrm{Pl}} \simeq 10^{19}$ GeV

  Planck Mass

set $\Gamma_{\mathrm{ann}}(T_{\mathrm{f}}) = H(T_{\mathrm{f}})$, and solve for T_{f}

Find: $x_{\mathrm{f}} = m/T_{\mathrm{f}} \sim \ln(m M_{\mathrm{Pl}} \sigma_1) \Rightarrow T_{\mathrm{f}} = m/x_{\mathrm{f}}$

So at freezeout

$$n_{\mathrm{f}} \sim \frac{x_{\mathrm{f}}^{3/2}}{m M_{\mathrm{Pl}} \sigma_1} T_{\mathrm{f}}^3 \tag{6}$$

→ present relic number density

$$n_{\psi,0} = n_{\psi,\mathrm{f}} \left(\frac{a_\mathrm{f}}{a_0}\right)^3 = n_{\psi,\mathrm{f}} \left(\frac{T_0}{T_\mathrm{f}}\right)^3 \sim \frac{x_\mathrm{f}^{3/2}}{m M_\mathrm{Pl} \sigma_1} T_0^3 \tag{7}$$

present relic mass density

$$\rho_{\psi,0} = m n_{\psi,0} \simeq \frac{x_\mathrm{f}^{3/2} n_{\gamma,0}}{M_\mathrm{Pl} \sigma_1} \tag{8}$$

What have we shown?
if a symmetric stable species ever created
(annihilates but not decays)
then annihilations will freeze, and
inevitably have nonzero relic density today.

This calculation is of the highest interest to particle physicists
Q: why?
We have calculated a relic density
Q: To what should this be compared?

Cold Relics: Present Abundance

★ $\rho_{\psi,0}$ indep of m_ψ

★ $\rho_{\psi,0} \propto 1/\sigma_1$: the weak prevail!
Q: what sort of cross section is relevant here?

★ To get "interesting" present density:
$\Omega_\psi \sim 1 \rightarrow \rho_\psi \sim \rho_{\rm crit}$ *demands* a *specific* cross section

$$\sigma_1 = \frac{x_{\rm f}^{3/2} n_{\gamma,0}}{\Omega_\psi M_{\rm p} \rho_{\rm crit}} \tag{9}$$

$$\sim 10^{-37}\ {\rm cm}^2 \left(\frac{x_{\rm f}}{10}\right)^{3/2} \tag{10}$$

scale of the Weak interaction! $[\sigma_{\rm weak}(E \sim {\rm GeV}) \sim 10^{-38}\ {\rm cm}^2]$

The WIMP Miracle

Dark Matter candidate:
if DM is a cold symmetric relic
needed *annihilation cross section* is at Weak scale!
corresponding energy: if $\sigma \sim \alpha/E^2$
then $\sigma \sim 10^{-36}$ cm^2 $=$ 10 pb $\rightarrow E \sim 1$ TeV

deeper reason for DM as
Weakly Interacting Massive Particle: **WIMP**

that weak-scale annihilations $\rightarrow \Omega_\chi \sim \Omega_{\mathrm{nbdm}}$: **"WIMP Miracle"**

How to find them?
What if we do? What if we don't?

Director’s Cut Extras

Freezeout and Relic Abundance of a Symmetric Species

for *conserved* species ψ (chem. pot. $\mu_\psi \neq 0$)

constant comoving number: $d(na^3) \overset{\text{cons}}{=} 0$

$$\Rightarrow \boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi \overset{\text{cons}}{=} 0}$$

for *non-conserved* species: $d(n_\psi a^3) = q a^3\, dt \neq 0$, where

q = *source/sink* rate = creation/destruction rate per unit vol

$$\Rightarrow \boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a}\, n_\psi = q}$$

assume annihilation $\psi\bar{\psi} \rightarrow X\bar{X}$ product X thermal,
with chem. pot. $\mu_X \ll T \ \Rightarrow\ n_X = n_{\bar{X}}$

$$\begin{aligned} q = q_{\text{net}} &= q_{\text{prod}} - q_{\text{ann}} && (11)\\ &= \langle\sigma v\rangle_{\text{prod}} n_X n_{\bar{X}} - \langle\sigma v\rangle_{\text{ann}} n_\psi n_{\bar{\psi}} && (12)\\ &= \langle\sigma v\rangle_{\text{prod}} n_X^2 - \langle\sigma v\rangle_{\text{ann}} n_\psi^2 && (13) \end{aligned}$$

in equilib, *Q: what condition holds for q?*

chem equilib: $q = 0 \Rightarrow \boxed{q_{\text{prod}} = q_{\text{ann}}}$
so in general

$$\dot{n}_\psi + 3Hn_\psi = q = -\langle\sigma v\rangle_{\text{ann}}\left[n_\psi^2 - (n_\psi^{\text{eq}})^2\right] \tag{14}$$

and a similar expression for $\bar{\psi}$

Change variables:
(1) use comoving coords:
photon density $n_\gamma \propto T^3 \propto a^{-3}$,
so put $Y = n_\psi/n_\gamma$ to remove volume dilution
then $\dot{n_\psi} + 3\dot{a}/a\; n_\psi = n_\gamma \dot{Y}$
(2) put $x = m_\psi/T \propto a$
since $t \propto 1/T^2 \propto x^2$,
$dY/dt = dY/dx\; \dot{x} = H\; x\; dY/dx$

Then:

$$Hx\frac{dY}{dx} = -n_\gamma\langle\sigma v\rangle_{\text{ann}}\left(Y^2 - Y_{\text{eq}}^2\right) \tag{15}$$

(16)

finally

$$\frac{x}{Y_{\text{eq}}}\frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right] \tag{17}$$

where $\Gamma_A = n_\psi^{\text{eq}}\langle\sigma v\rangle_{\text{ann}}$: annihil. rate

So: change in comoving ψ controlled by
(1) annihil. effectiveness Γ/H
(2) deviation from equil

when $\Gamma/H \gg 1$

Q: what if $Y < Y_{\text{eq}}$? $Y > Y_{\text{eq}}$?

when $\Gamma/H < 1$

Q: how does Y change?

Q: how you you expect Y to evolve?

when $\Gamma/H \gg 1$, Y driven to Y_{eq}
when $\Gamma/H < 1$, Y change is small → freezeout!

relic abundance at $T \to 0$ or $x \to \infty$ is
$Y_\infty \simeq Y_{\mathsf{eq}}(x_{\mathsf{f}})$: value at freezeout

Step back:
How can a symmetric species have
$n_\psi = n_{\bar{\psi}} \neq 0$ at $T \ll m$?

physically: expansion is key

if $H = 0$, $Y_\infty = Y_{\rm eq}(\infty) = 0$:

$\rightarrow$ all ψ find $\bar{\psi}$ partner, annihilate

but $H \neq 0$: when U dilute enough,

ψ never finds $\bar{\psi}$: i.e., $\Gamma \ll H$

nonzero relic abundance

hot relics: $x_{\rm f} \ll 1$ $(T_{\rm f} \gg m)$

cold relics: $x_{\rm f} \gg 1$

Note: hot/cold *relics* refers to freezeout conditions

But: hot/cold *dark matter* refers to structure formation criteria

(namely, m vs temp $T_{\rm eq}$ at matter-rad equality)