Astro 596/496 NPA
Lecture 26
April 1, 2019

Announcements:

- **Preflight 5 due Friday**

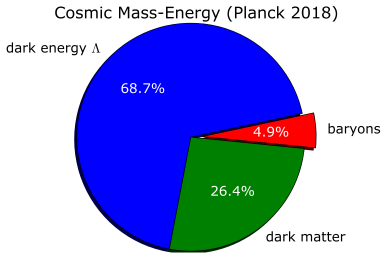
Last time:

Finished Early Universe, nuclear and particle cosmology sets the stage for the rest of cosmic time

Taking Stock:

Cosmic Pie

Q: where do we think the slices come from?



`www: Solar Abundances`

Q: what features do we now understand? what remains?

STELLAR EVOLUTION AND NUCLEOSYNTHESIS

Stellar Evolution and Nucleosynthesis

Overview

Star structure, evolution, nucleosynthesis
essentially determined by stellar:

- mass
- composition
- other factors if present: binarity, spin, magnetism

theory: for non-rotating, non-magnetic stars
inputs: mass M, composition determine
outputs: structure and evolution; history of
L, T_{eff}, τ, nucleosynthesis

recall:
stellar lifetimes $\tau(M)$ very strongly *inverse* with mass
Q: implications for stellar populations and nucleosynthesis?

Stellar Lifetimes and Nucleosynthesis Roles

mass M	lifetime $\tau(M)$	fate
$\lesssim 0.9 M_\odot$	$\gtrsim t_0$	"never" die
1 to $\sim 8 M_\odot$	10 Gyr to 30 Myr	red giant → AGB → white dwarf + PN
$\gtrsim 8 M_\odot$	$\lesssim$ 30 Myr	supernova, direct collapse(?)

- low-mass stars just "accumulate"
  $\Rightarrow$ "sinks" for baryons and nucleosynthesis products
- high-mass stars rapidly die:
  → first sources of post-big bang elements
  → many supernova "generations" till today
- different nucleosynthesis roles for different masses

Solar Models

Charity begins at home: *understand the Sun first*

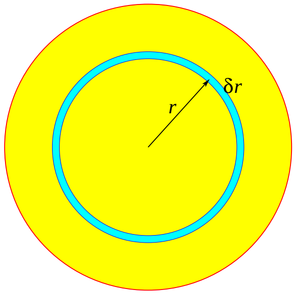
focus on **solar neutrinos**:
new physics, powerful diagnostic of solar/stellar models

modeling the Sun and its neutrino production:
Basic Assumptions? Ingredients?

Solar Model Ingredients (ASTR 404, 504)

1. **Hydrostatic equilibrium**: *pressure–gravity balance*
consider spherical shell of width dr,

volume $dV = 4\pi r^2 dr$



weight: $mg = \rho dV \; Gm(r)/r^2 = 4\pi G m(r) \rho dr$ (down)

pressure diff: $P_{\rm net} = -P(r+dr) + P(r) = -dP/dr \; dr$

$\Rightarrow$ *force*: $F_p = P_{\rm net} A = -4\pi r^2 P_{\rm net}$ (up)

force balance:

$$-\frac{dP}{dr} = \frac{Gm(r)\rho}{r^2} \qquad (1)$$

using $dm(r)/dr = 4\pi r^2 \rho(r)$ (Lagrangian "mass coordinate")

$$-\frac{dP}{dm} = \frac{Gm dm/dr}{4\pi r^4} \qquad (2)$$

Equation of State: $p = \rho k T/m + aT^4/3$

Q: this accounts for momentum; other conditions/equations?

2. **Energy conservation and transport:**

Center: energy transport via radiation

Envelope: transport via convection (recall ^{7}Li depletion)

energy loss is via photons & is diffusive.

photon *energy flux* is

$$F = \langle c p_r v_r n_\gamma \rangle = \frac{1}{3} c \varepsilon_\gamma = \frac{ca}{3} T^4 \tag{3}$$

(where $\varepsilon_\gamma = aT^4$)

net flux at r: $F_{\rm net} = F(r + \delta r) - F(r) \simeq dF/dr \ \delta r$

diffusion: "stepsize" δr is scatter mfp $\lambda = 1/n\sigma \equiv 1/\rho\kappa$

scattering opacity $\kappa = \sigma n/\rho = \sigma/m$

local luminosity: $\ell = 4\pi r^2 F_{\rm net}$

$$\frac{\ell}{4\pi r^2} = \frac{1}{\rho\kappa}\frac{dF}{dr} = \frac{4acT^3}{3\rho\kappa}\frac{dT}{dr} \tag{4}$$

Q: for nuclear reaction $ab \to cd$*: what is rate per vol?*

Q: what is energy production per vol?

3. **Energy generation via nuke reactions**

put $\rho\epsilon$ = nuke energy production rate per unit vol

$$\begin{aligned} d\ell &= \rho\epsilon\ dV = \rho\epsilon\ n 4\pi r^2\ dr \tag{5} \\ \frac{d\ell}{dr} &= 4\pi r^2 \rho\epsilon \tag{6} \end{aligned}$$

for nuclear reaction $a+b \rightarrow c+d$

$\dot{n}_{ab\rightarrow cd} = \langle \sigma_{ab} v \rangle n_a n_b$

$\qquad$ = nuke reaction rate per volume

so $\rho\epsilon = Q\dot{n}$

where energy release $Q = \Delta_a + \Delta_b - \Delta_c - \Delta_d$

Now have differential equations

but still need one more thing to solve them

∞ *What's that?*

4. **Boundary conditions:** the present-day Sun

$t_\odot = 4.6$ Gyr

$M_{\text{tot}} = M_\odot = 2.0 \times 10^{33}$ g

$R = R(t_\odot) = R_\odot = 7.0 \times 10^{10}$ cm

$L = L_\odot = 3.8 \times 10^{33}$ erg/s

With these, solve $m(r)$, $\ell(m)$, $T(m)$ (vs time)

method: start at Sun center, assume ρ_c, T_c

integrate outward *Q: until?*

adjust central conditions until match present Sun

our focus: nuclear reactions. depends on ρ_c, T_c

Back of the Envelope

Order of magnitude:

$$\frac{dP}{dR} \sim \frac{P_c}{R} \tag{7}$$

$$\sim \frac{GM\rho}{R^2} \tag{8}$$

ideal gas: $P = \rho kT/m$

$$T_c \sim \frac{(m_p/2)P_c}{\rho k} = \frac{GMm_p}{2kR} \sim 10^7 \text{ K} \sim 1 \text{ keV} \tag{9}$$

Why $m_p/2$*?*

Q: is this hot or cold?

now compare to professional result...

Standard Solar Model: Bahcall & Pinsonneault (2000,2004)

predictions for present-day conditions at solar center:

$$\begin{aligned} T_c &= 1.57 \times 10^7 \text{ K} && (10) \\ \rho_c &= 152 \text{ g cm}^{-3} && (11) \\ X_c = \left(\frac{\rho_\mathrm{H}}{\rho_\mathrm{B}}\right)_c &= 0.34 && (12) \\ Y_c = \left(\frac{\rho_\mathrm{He}}{\rho_\mathrm{B}}\right)_c &= 0.64 && (13) \end{aligned}$$

Sun is in **hydrogen "burning" phase: main sequence**
a series ("chain") of reactions with net effect

$$4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e$$

Q: reaction steps?

Solar Hydrogen Burning: Big Picture

Sun: main sequence, $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

Reaction chains usually begin with

$$p + p \rightarrow d + e^+ + \nu_e \qquad (14)$$

weak reaction: slowest step in chain. Next:

$$d + p \rightarrow {}^3\text{He} + \gamma \qquad (15)$$

Then: 3 branches

PP-I

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p$$

$${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$$

PP-II

$${}^7\text{Be} + e \rightarrow {}^7\text{Li} + \nu_e$$

$${}^7\text{Li} + p \rightarrow {}^4\text{He} + {}^4\text{He}$$

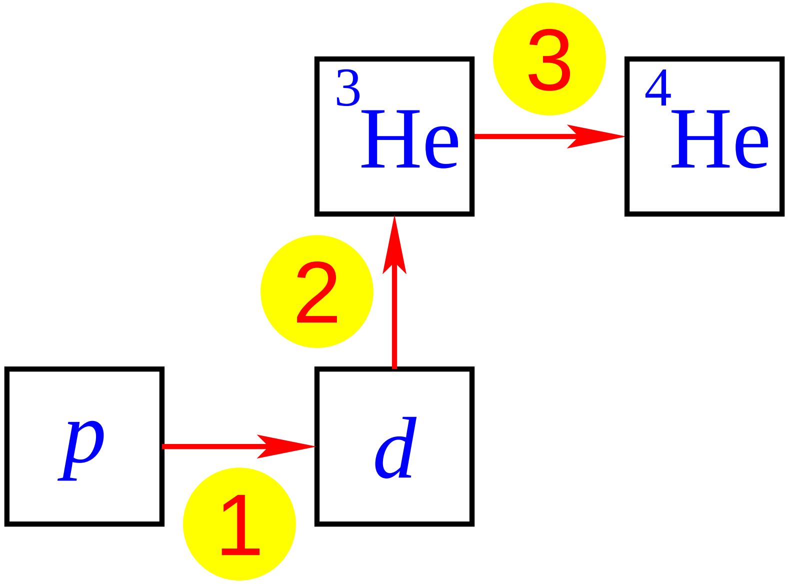
PP-III

$${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$$

$${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$$

$${}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$$

PP–I Chain



① $pp \rightarrow de\nu$

② $dp \rightarrow {}^3\mathrm{He}$

③ $^3\mathrm{He}\ {}^3\mathrm{He} \rightarrow {}^4\mathrm{He}\ pp$

The PP-I Chain

Deuterium

d source $pp \rightarrow de^{+}\nu_e$, rate per vol $\lambda_{pp} n_p^2/2$

d sink: $dp \rightarrow {}^3\mathrm{He}\gamma$, rate $\lambda_{dp} n_d n_p$

evolution of d:

$$\begin{aligned} \dot{n}_d &= -\lambda_{dp} n_d n_p + \lambda_{pp} n_p^2/2 && (16) \\ &= -\Gamma_{\mathrm{per}\,d}(dp \rightarrow {}^3\mathrm{He}\gamma) \left(n_d - n_d^{\mathrm{eq}}\right) && (17) \end{aligned}$$

where $\Gamma_{\mathrm{per}\,d} = n_p \langle \sigma v \rangle_{dp \rightarrow {}^3\mathrm{He}\gamma} \equiv n_p \lambda_{dp}$

Q: what is n_d^{eq} mathematically? physically?

Q: what is physical significance of $\Gamma_{\mathrm{per}\,d}$?

Q: consequence for D/H ratio in Sun?

Deuterium in the Sun

deuterium evolution in Sun:

$$\dot{n}_d = -\lambda_{dp} n_d n_p + \lambda_{pp} n_p^2/2 \quad (18)$$

$$= -\Gamma_{\mathrm{per}\,d}(dp \rightarrow {}^3\mathrm{He}\gamma) \left(n_d - n_d^{\mathrm{eq}}\right) \quad (19)$$

self-regulating:

driven to equilibrium $\dot{n}_d = 0$

in timescale $\tau_d = 1/\Gamma_{\mathrm{per}\,d} \sim 1$ s (!)

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{eq}} = \left(\frac{n_{d,\mathrm{eq}}}{n_p}\right) = \left.\frac{\lambda_{pp}}{2\lambda_{dp}}\right|_{T_c} \sim 10^{-18} \quad (20)$$

why so small? ratio of Weak to EM reaction

note: forward $pp \rightarrow de^+\nu_e$ has proton consumption rate

$\dot{n}_p|_{pp\rightarrow de\nu} = -\lambda_{pp} n_p^2 = -2\Gamma_{\mathrm{per}\,p} n_p$ *Q: why?*

Q: what happens to ^{3}He*? possible reactions?*