Astro 596/496 NPA

Lecture 29

April 10, 2019

Announcements:

- **Problem Set 5 due Friday**
- Office hours after class today

Event Horizon Telescope has imaged M87 black hole!

- shadow detected, and asymmetric ring
- direct confirmation of black hole event horizon
- asymmetry $\rightarrow$ doppler boost of accretion disk $\rightarrow$ spin direction
- $M = (6.5 \pm 0.2_{\rm stat} \pm 0.7_{\rm sys}) \times 10^9 M_\odot$ agrees with stellar dynamics
- Illinois a key player! Prof. Charles Gammie led simulation effort with grad students Ben Ryan, George Wong, Ben Prather
- Gammie Astronomy Colloquium: April 30 arrive early!
- party like it's 1999! this doesn't happen every day!

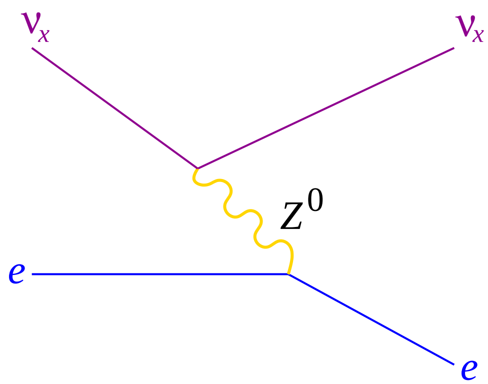
Last Time: The Case of the Missing Neutrinos

more on neutrino physics: neutral vs charged current interactions
Q: what's that? differences? similarities?

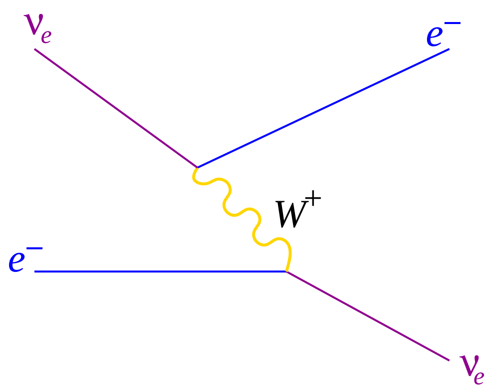
solar neutrino problems
Q: what are they? what do they suggest? why was SNO crucial?

solar neutrino solution: new neutrino physics
Q: what's the basic idea?

Neutral vs Charged Current



neutral current events
occur for all ν (and $\bar{\nu}$) flavors
with equal probability



charged current events
only have e^- targets (no ambient μ or τ!)
occur for only for ν_e!

ν_e, ν_μ, ν_τ *flavor* set by charged lepton partner
ω so in Weak interaction: ν_e produced with e, and ν_μ with μ, etc.
that is: W couples ν_e and e, ν_μ and μ, etc.

Neutrino Oscillations in Vacuum: The Quantum Neutrino

If *neutrinos have nonzero mass*

- family status (e, μ, τ "**flavor**"), and
- **mass**

can be **distinct**!

ν family $\rightarrow$ lepton number conservation in Weak interactions

formally, νs couple to Weak interaction as

flavor eigenstates

flavor basis vectors $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$

free (vacuum) neutrino $\rightarrow$ *propagates* as

mass eigenstate

mass basis vectors $|j\rangle$, $j = 1, 2, 3$

Neutrino Oscillations: Spin-1/2 Analogy

consider a *beam of electrons*, with spin $s = 1/2$: 2 states

create with *spin up*
$s_z = +1/2$ wavefunction $|\text{init}\rangle = |\uparrow\rangle$

propagate through magnetic field in x axis
spin rotated an angle θ with respect to z

observe with detector aligned in z
measure wavefunction $|\text{obs}\rangle$ spin-up and spin-down components
i.e., $\langle\uparrow|\text{obs}\rangle$ and $\langle\downarrow|\text{obs}\rangle$
infer *probability* $P(\theta) = P(\uparrow_{\text{init}}, \uparrow_{\text{obs}})_\theta = \|\langle\uparrow|\text{obs}\rangle\|^2$
of observing in spin-up state

Q: what is $P(\theta = 0)$*?* $P(\theta = \pi)$*?* $P(\theta = \pi/2)$*?* $P(\theta)$*?*
Q: what's going on physically when $P < 1$*?*

Spin-1/2 Analogy

for 2-state system like spin-$\frac{1}{2}$: two eigenstates

in z basis: $s_z = \pm 1/2$ eigentstates are

$$|\uparrow\rangle = \psi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |\downarrow\rangle = \psi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

in this basis, the x-axis $s_x = \pm 1/2$ eigent states are

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \quad , \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{|\uparrow\rangle - |\downarrow\rangle}{\sqrt{2}}$$

but these also form a perfectly valid basis

- lesson: *eigenstates of one basis are linear combo of other basis*

Basis Transformation: Flavor/Weak $\leftrightarrow$ Mass/Vacuum

neutrino **mass eigenstate $\neq$ flavor eigenstate**

either basis a valid description of ν state

physical situation selects most natural choice:

- ν *production/detection*: Weak interaction $\longrightarrow$ *flavor* basis
- ν *propagation* in vacuum $\longrightarrow$ *mass* basis

Q: what does this mean for solar neutrinos?

Neutrino Flavor Change

Key idea:

- neutrinos *born* in Weak interactions
  $\rightarrow$ *created* as *Weak* eigenstates
- *propagate* as *vacuum* eigenstates
- then *detected* in *Weak* interactions

Evolution of wavefunction during propagation
changes probability of remaining a ν_e state

If mass eigenstates have definite p and thus $E_j = \sqrt{p^2 + m_j^2}$ (as in vacuum), then Schrödinger:

$$i\hbar \frac{d}{dt} |\nu_{\text{mass}}\rangle_j = H_{\text{vacuum}} |\nu_{\text{mass}}\rangle_j = E_j |\nu_{\text{mass}}\rangle_j \tag{2}$$

and so

$$|\nu_{\text{mass}}(t)\rangle_j = e^{-iE_j t/\hbar} \; |\nu_{\text{mass}}(0)\rangle_j \tag{3}$$

Two flavors: allow 2 flavors (e and x) to mix
write $|f\rangle = U_{\text{vac}}|m\rangle$, where

$$U_{\text{V}} = \begin{pmatrix} \cos\theta_{\text{V}} & \sin\theta_{\text{V}} \\ -\sin\theta_{\text{V}} & \cos\theta_{\text{V}} \end{pmatrix} \tag{4}$$

with vacuum mixing angle $\theta_{\text{V}} \in (0, \pi/4)$ ("ν_e mostly ν_1")

$$|\nu_e(t)\rangle = e^{-iE_1 t/\hbar}\cos\theta_{\text{V}}|1\rangle + e^{-iE_2 t/\hbar}\sin\theta_{\text{V}}|2\rangle \tag{5}$$

where E_1, E_2 have same momentum p

Solar neutrinos start ($t = 0$) as pure ν_e
QM **amplitude** at t to *remain* ν_e:

$$\langle\nu_e(0)|\nu_e(t)\rangle = e^{-iE_1 t/\hbar}\cos\theta_{\text{V}}^2 + e^{-iE_2 t/\hbar}\sin\theta_{\text{V}}^2 \tag{6}$$

$\Rightarrow$ probability to remain ν_e:

$$|\langle\nu_e(0)|\nu_e(t)\rangle|^2 = 1 - \sin^2 2\theta_{\text{V}}\ \sin^2\left[1/2\ \frac{(E_2 - E_1)t}{\hbar}\right]$$

Since $m(\nu_i) \ll p$, $E_j = \sqrt{p^2 + m_j^2} \simeq p + m_j^2/2p$, and

$$E_2 - E_1 \simeq \frac{m_2^2 - m_1^2}{2E} = \frac{\pm\Delta m^2}{2E} \tag{7}$$

$\Delta m^2 = |m_2^2 - m_1^2| > 0$

E = avg energy.

In time t go distance $L \simeq ct$

$$\begin{aligned} P(\nu_e^{\text{birth}} \rightarrow \nu_e^{\text{detect}}) &= |\langle \nu_e(0)|\nu_e(t)\rangle|^2 \\ &= 1 - \sin^2 2\theta_{\text{V}} \sin^2\left(\pi \frac{L}{L_{\text{V}}}\right) \qquad (8) \\ &= 1 - \sin^2 2\theta_{\text{V}} \sin^2\left[1.27 \frac{\Delta m^2(\text{eV}^2) L(\text{km})}{E(\text{GeV})}\right] \end{aligned}$$

where $L_{\text{V}} = 4\pi\hbar E/\Delta m^2$ "vacuum oscillation length"

$$P(\nu_e^{\text{birth}} \to \nu_e^{\text{detect}}) = |\langle \nu_e(0) | \nu_e(t) \rangle|^2 = 1 - \sin^2 2\theta_{\text{V}} \; \sin^2 \left(\frac{\pi L}{L_{\text{V}}} \right)$$

Minimum mass sensitivity: $\pi L / L_{\text{V}} = \pi/2$
If $L_{\text{V}} \ll 1$ AU: wash out differences among species
If $L_{\text{V}} \simeq 1$ AU: solve solar ν problem!

$$\Delta m^2 \sim 10^{-12} \text{ eV}^2 \left(\frac{E}{10 \text{ MeV}} \right) \tag{9}$$

solves solar ν problem, but dubious
Q: why?
$\Rightarrow$ "just-so" solution

also note: if Δm^2 larger, $L_V \ll 1AU$

$$\Rightarrow \quad |\langle \nu_e(0) | \nu_e(t) \rangle|^2 \simeq 1 - \frac{1}{2} \sin^2 2\theta \geq \frac{1}{2} \tag{10}$$

but we need suppression $> 50\%$!
can't do this with vacuum oscillations!

Neutrino Oscillations in Matter

MSW = Mikheyev, Smirnov, Wolfenstein

νs pass thru matter twice (in Sun, in Earth)
all ν types can have NC interactions
but ν_e have extra CC interactions ($\nu e \rightarrow \nu e$)
selectively modifies ν_e flux

ν_e potential in matter: $V_e(r) = \sqrt{2}\, G_\mathsf{F} n_e(r)$
put $\langle \nu_e(0) | \nu_e(t) \rangle = c_e(t)$, similar $c_x(t)$
Schrödinger equation + algebra:

$$i\hbar \frac{d}{dt} \begin{pmatrix} c_e \\ c_x \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\theta_\mathsf{V} + 2\sqrt{2}\, G_\mathsf{F} n_e E & \Delta m^2 \sin 2\theta_\mathsf{V} \\ \Delta m^2 \sin 2\theta_\mathsf{V} & \Delta m^2 \cos 2\theta_\mathsf{V} - 2\sqrt{2}\, G_\mathsf{F} n_e E \end{pmatrix} \begin{pmatrix} c_e \\ c_x \end{pmatrix}$$

Q: evolution as $n_e \rightarrow \infty$? $n_e \rightarrow 0$?
Q: condition for maximal mixing?
Q: so how will ν states evolve when propagating from solar core?

maximal mixing ("resonance") when diagonal elements zero:
$\rightarrow 2\sqrt{2}\, E G_{\mathsf{F}} n_e = \Delta m^2 \cos 2\theta_{\mathsf{V}}$: density-dependent!

$$\begin{aligned} m_u n_e^{\mathsf{crit}} &= \frac{m_u \Delta m^2 \cos 2\theta_{\mathsf{V}}}{2\sqrt{2} G_{\mathsf{F}} E} \\ &= 66 \text{ g cm}^{-2} \cos 2\theta_{\mathsf{V}} \left(\frac{E}{10 \text{ MeV}}\right)^{-1} \left(\frac{\Delta m^2}{10^{-4} \text{ eV}^2}\right) \end{aligned}$$

Can happen in Sun! No fine tuning needed!

- start as ν_e, in dense region where $n_e > n_e^{\mathsf{crit}}$
  neutrinos leave, seeing a dropping electron density
- reach $n_e = n_e^{\mathsf{crit}}$ $\rightarrow$ *change to* ν_x
- continue to Earth

works for range of Δm^2 *Q: how?*

But note energy dependence:
Q: what energies, ν *populations, experience MSW?*

Solar Neutrino Solutions

Using all solar ν data, most favored solution:
⋆ $\theta_V = 32.5^\circ$
⋆ $\Delta m^2 = 7.1 \times 10^{-5}$ eV^2

Implications

- "large mixing angle" (LMA)
  Q: what angle gives maximal vacuum mixing? ...hint:

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- $\Delta m^2 = |m_2^2 - m_1^2|$ does *not* give either m_1 or m_2
  but does set *minimum* mass for either:
  $m_{\nu,\text{min}} = \sqrt{\Delta m^2} = 8 \times 10^{-3}$ eV

Q: how to test this solution in the lab?

Laboratory test: KamLAND

(Kamiokande Liquid Scintillator Anti-Neutrino Detector)
sources: anti-neutrinos from Japanese nuke reactors

- $E_\nu = 2.6 - 8$ MeV
- avg distance $R \sim 180$ km

$\rightarrow$ if LMA, disappearance probability is

$$P_{\rm dis} = \sin^2 2\theta_{\rm V} \; \sin^2 \left(2\pi \frac{R}{350{\rm km}} \right) \tag{11}$$

Kamland observes flux *reduction*: $P_{\rm dis} = 0.66$
E_ν spectrum $\rightarrow \Delta m^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5}$ eV^2
$\rightarrow$ confirms oscillations in general, and LMA in particular!
`www: KamLAND plots`

Solar Neutrino Problem Solved!

Q: remaining questions? experiments?

Next Step: Precision Neutrino Astronomy

- measure monoenergetic ^{7}Be neutrinos
  now detected in real-time!
  flux consistent with MSW LMA
  `www: Borexino`
- measure pp flux to $\sim 1\%$ $\Rightarrow$ better θ_{V}
  `www: Stanford Lab`

New questions:
What are ν masses?
oscillations only measure splittings Δm^2
$\longrightarrow$ know masses are *different* and *nonzero*
but don't even know hierarchy: is $m_1 < m_2$ or the reverse?

Is ν_i identical to $\bar{\nu}_i$?

yes: "Majorana" neutrinos

no: "Dirac" neutrinos, right-hand ν exist

can test with "neutrinoless double beta decay"

(rare nuclear decays, only go if Majorana)

Do neutrinos violate CP?

if so: maybe important in baryogenesis...

"leptogenesis" scenario: generate net *lepton* number, then translate this to net baryon number

Director's Cut Extras

Three-Flavor Mixing

Full neutrino description has three flavor states
and thus three mass states

basis vectors related by linear transformation

(P)MNS=Pontecorvo, Maki, Nakagawa, Sakata matrix

$$|\nu_{\mathrm{flavor}}\rangle_{i\in e,\mu,\tau} = \sum_{j=1,2,3} U_{ij}|\nu_{\mathrm{mass}}\rangle_j \tag{12}$$

$$|\nu_{\mathrm{mass}}\rangle_{i\in 1,2,3} = \sum_{j=e,\mu,\tau} U^{\dagger}_{ij}|\nu_{\mathrm{flavor}}\rangle_j \tag{13}$$

U is time-indep, unitary: $U^{-1} = U^{\dagger}$; $U^{\dagger}U = UU^{\dagger} = 1$