

PRINT Last Name _____**PRINT** First Name _____

Net ID _____

Signature _____

Instructions- This is a closed book, closed notes exam. You have 1.5 hours to complete it.

- **Print your last and first name, then fill in your Net ID, and signature.**
- **At the end of this exam, you must return this exam with all pages, and you must return your scantron sheet. Please circle all of your answers on this exam and fill in all of your answers on the scantron.**
- **If you do not turn in a complete exam and scantron form, you will receive the grade AB (Absent) for this exam.**
- **Use a #2 pencil. Each question has only *one* answer.** If you bubble in more than one answer it will automatically be marked wrong. Erase mistakes completely.
- This exam is **either Form A, B, C.** You don't know which test form you have so you **MUST** turn in your scantron with the exam so the TAs can correctly mark the test form box on your scantron sheet after the exam.

READ → How to fill out the Scantron form

Print and bubble in your **LAST NAME** with **NO SPACES or DASHES** starting in the left most column. Print your **FIRST INITIAL** in the right-most column.

- Print and bubble in your Student ID number (UIN) **NO SPACES or DASHES** in the Student Number box.
- Print and bubble in the date in the Date box.
- ****Print and bubble in your NET ID with NO SPACES or DASHES in the NETWORK ID box. ** (Automatic 1 point deduction if you don't bubble in your NET ID correctly.)****
- Print and bubble in the Section Box. See section codes →.
- *Write Stat 200* on the COURSE line.
- *Write your instructor's name* on the INSTRUCTOR line.
- *Write your section on the SECTION line.*
- **Sign your name, and right underneath the student signature line PRINT your name**

Section Codes:

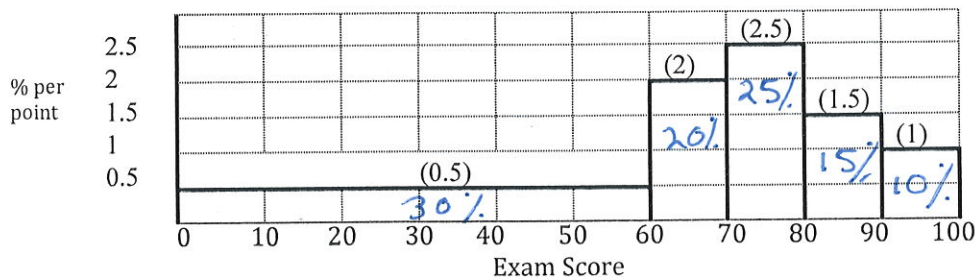
L1 (Fireman) = 00001
 ONL (Fireman) = 00002
 S1 (Liu) = 00003
 S2 (Wang) = 00004
 S3 (Yubai) = 00005
 S4 (Yang) = 00006
 S5 (Chen) = 00003

Warning -All Cheating including being caught with a non-permissible calculator or formula sheet will result in a 0 and an academic integrity violation on your University record.

CHECK NOW THAT YOU HAVE COMPLETED ALL OF THE STEPS. Before starting the exam, check to make sure that your test booklet is complete. You should have **11 pages (73 problems)**, including **3 tables**: the normal table, the *t*-table, and the chi-square table.

Question 1-7 pertain to the histogram below.

The histogram below depicts the exam scores of 500 students. The height of each block is given in parentheses. (Assume even distribution throughout each interval.)



- What percent of the students scored in the 70-80 block? a) 10% b) 15% c) 20% **d) 25%** e) 30%
- The median is closest to ... a) 40 b) 50 c) 60 **d) 70** e) 85
- The median is > the average. a) less than **b) greater than** c) equal to d) cannot be determined
- The 30th percentile is **a) 60** b) 70 c) 75 d) 80 e) 90
- If 5 points were added to each of the 500 students' exam scores, the average would
 - Increase by 5%
 - Increase by 0.5 points
 - c) Increase by 5 points**
 - Stay the Same
 - Decrease
- and the SD would
 - Increase by 5%
 - Increase by 0.5 points
 - Increase by 5 points
 - d) Stay the Same**
 - Decrease
- If you knew the average and SD of the exam scores, would it be appropriate to use the normal approximation to figure what percentage of the exam scores fell within various intervals?
 - Yes, because 500 is a large sample size.
 - Yes, because the exam scores range from 0 to 100.
 - c) No, because the histogram of the exams scores does not look at all like the normal curve.**
 - Maybe, depending on how the students were selected to take the exam.

The next 3 questions pertain to a list of 4 numbers with the following deviations from the average: 2, 2, -2, -2

- One of the deviations is missing, what is it? a) 0 b) -1 **c) -2** d) 1 e) 2 *deviations sum = 0*
- What's the SD of the 4 numbers? a) 0 b) -1 c) -2 d) 1 **e) 2** $\sqrt{\frac{2^2 + 2^2 + (-2)^2 + (-2)^2}{4}} = 2$
- If all 4 numbers on the list were multiplied by -2, the SD would...
 - a) be multiplied by 2**
 - be multiplied by -2
 - stay the same
 - be multiplied by either 2 or -2 depending on whether the numbers on the list were positive or negative.

The next 5 questions pertain to this study: Does the difficulty of the first question on an exam affect exam performance? To find out researchers randomly divided 1000 high school seniors into 2 groups. Both groups were given the same exact math test under the same exact conditions, the only difference was that Group A's exam started with a difficult question and Group B's exam started with an easy question. The exam was machine graded and both the proctors and the students thought all the exams were the same. Group A did worse and the results were highly significant.

- 11) Based only on the information above, this study is an example of
- a) Observational Study
 - ☒ b) Randomized controlled double-blind experiment.
 - c) Non-Randomized Controlled Experiment
- 12) Can we conclude that it was the difficulty of the first question on the exam that *caused* the difference in the exam scores?
- a) No, correlation is not causation. There are many factors that affect exam performance including the demographics of the students and the conditions under which they took the exam.
 - ☒ b) Yes, because the groups were randomly assigned so systematic differences between them were removed.
 - c) Yes, highly significant results allows us to conclude causation even in non-randomized studies.
- 13) Which of the following are likely to confound the results?
- a) Time management: Students may have spent too much time on the first problem and didn't have enough time to finish the exam.
 - b) Anxiety: Seeing a difficult problem first may have caused students to get anxious resulting in lower test scores.
 - c) Study habits: Those who studied well ahead of time were prepared and not thrown off by seeing a difficult question first.
 - d) All of the above are likely confounders.
 - ☒ e) None of the above are likely confounders.
- 14) Suppose 30 of the 1000 students in the study have math learning disabilities. The researcher wanted to make sure those 30 students were evenly assigned to Group A and Group B but don't want to introduce bias. What should they do?
- Block
- ☒ a) At the start of the study they should divide the students into 2 groups (those with math learning disabilities and those without) and then randomly divide half of each group to A and half to B.
 - b) They should randomly assign half of the 1000 students to Group A and half to Group B. This will ensure that groups will be evenly divided on all characteristics including math learning disabilities.
 - c) There is no way they can ensure that exactly 15 of the 30 will end up in Group A and exactly 15 in Group B without introducing bias.
- 15) Many students never finished the exam so they didn't get to the last question. What should the researchers do?
- a) Break the students into 2 groups: those who finished and those who didn't, and compare the exam scores of those who finished in Group A to those who finished in Group B.
 - ☒ b) Compare everyone in Group A to everyone in Group B whether they finished the exam or not.

The next 5 questions pertain to the following:

Observational

Does taking the Stat 100 exam earlier in the day improve scores? To find out we compared the average exam scores of 1000 students who had chosen to take the Stat 100 conflict exam in the afternoon to the 4000 students who had chosen to take the regular exam at night and found the students who took the earlier exam did significantly better.

16) Which of the following statements is best? *Choose one:*

- a) This is strong evidence that taking the exam earlier *caused* higher test scores.
- ☒ b) This only shows that taking the earlier exam is *associated with* higher exam scores: it doesn't show whether or not the earlier time of day actually *caused* higher test scores.
- c) This shows that taking the exam earlier is *associated with but could not possibly cause* higher test scores.

Identify whether the following are confounders, causal links, or neither.

- 17) Alertness- Students are more alert earlier in the day, which helps them perform better on exams.
 a) Confounder ☒ b) Causal Link c) Neither *Early Exam → [Alert] → Higher Scores*
- 18) More prepared - Students who are more prepared sign up to take the exam earlier and are also more likely to do better on exam.
☒ a) Confounder b) Causal Link c) Neither *Early Exam ← [More Prepared] → Higher Scores*
- 19) Math anxiety- Math anxiety can lead to lower exam scores.
 a) Confounder b) Causal Link ☒ c) Neither *Early Exam? [Math Anxiety] → Lower Scores*
- 20) If we thought that gender might be a confounder what should we do to eliminate its confounding effect?
 a) Compare male scores to female scores within each exam time group.
☒ b) Compare the scores of males taking the conflict to males taking the regular exam. Do the same with females.
 c) Combine the afternoon and night exams and compare the overall male exam average to female exam average.

The next 3 questions pertain to the following: (Use the normal table at the end of this exam to answer these questions.)
 Assume IQ scores are normally distributed with an average = 100 and a SD = 20

21) About what percentage of the population have IQ's over 105?

- a) 4% b) 8% c) 20% ☒ d) 40% e) 50%

22) Someone in the 60th percentile has an IQ of

- a) 105 ☒ b) 110 c) 114 d) 118

23) What percent of the population have IQ's between 120 and 140?

- a) 10% ☒ b) 13.5% c) 17% d) 20% e) 24%

The following 5 questions pertain to rolling fair dice

24) One die is rolled 3 times. What's the chance of getting ~~no~~ 2's?

- a) $(5/6)^3$ ☒ b) $(1/6)^3$ c) $1 - (5/6)^3$ d) $1 - (1/6)^3$ e) $3/6$

25) One die is rolled 3 times. What's the chance of getting ~~at least one~~ 2?

- a) $(5/6)^3$ b) $(1/6)^3$ c) $1 - (5/6)^3$ ☒ d) $1 - (1/6)^3$ e) $3/6$

26) One die is rolled 3 times. What's the chance that ~~not all~~ 3 rolls are 2's?

- a) $(5/6)^3$ b) $(1/6)^3$ ☒ c) $1 - (5/6)^3$ d) $1 - (1/6)^3$ e) $3/6$

27) One die is rolled twice. What's the chance of getting a 3 on the first roll **and** a 4 on the second roll.

- ☒ a) $1/36$ b) $2/36$ c) $6/36$ d) $11/36$ e) $12/36$

28) One die is rolled twice. What's the chance of getting a 3 on the first roll **or** a 4 on the second roll.

- a) $1/36$ b) $2/36$ c) $6/36$ ☒ d) $11/36$ e) $12/36$

$$\frac{1}{6} + \frac{1}{6} - \frac{1}{36}$$

The next 2 questions pertain to routine screening for prostate cancer. If a man has prostate cancer there's a 90% chance the test will correctly give him a positive result and if a man doesn't have prostate cancer there's an 80% chance he'll correctly get a negative result. Suppose only 10% of those who get tested really have prostate cancer.

| | Tests Negative | Tests Positive | Total |
|-------------------------------|----------------|----------------|-------|
| Has Prostate Cancer | 1 | 9 | 10 |
| Does not have Prostate Cancer | 72 | 18 | 90 |
| Total | 73 | 27 | 100 |

29) What's the probability of having prostate cancer given a negative test result.

- (a) 1/73 b) 1/10 c) 9/27 d) 8/10 e) 72/73

30) What's the probability of having prostate cancer given a positive test result?

- a) 1/73 b) 1/10 (c) 9/27 d) 8/10 e) 72/73

The next 4 questions pertain to this situation:

A 100-question true/false test awards 1 point for each correct answer and **subtracts 1 point** for each incorrect answer.

31) Suppose a student guesses at random on each question what is the corresponding box model?

- (a) It has two tickets: 1 and -1
 b) It has two tickets: 1 and 0
 c) It has 100 tickets marked 1 and 0. The exact percentages of each is unknown and estimated from our sample.
 d) It has 100 tickets marked 1 and -1. The exact percentages of each is unknown and estimated from our sample.

$$\boxed{1} \quad \boxed{-1} \quad \text{avg} = 0 \quad \text{SD} = |1 - (-1)| \sqrt{\frac{1}{2} \cdot \frac{1}{2}} = 1$$

32) The expected value of the student's score is... (a) 0 b) 10 c) 20 d) 40 e) 50

$$E V_{\text{sum}} = n \cdot \text{avg} = 100 \cdot 0 = 0$$

33) The SE of the student's score is (a) 10 b) 5 c) 1 d) 0.1 e) not enough info

$$SE_{\text{sum}} = \text{SD} \sqrt{n} = 1 \sqrt{100} = 10$$

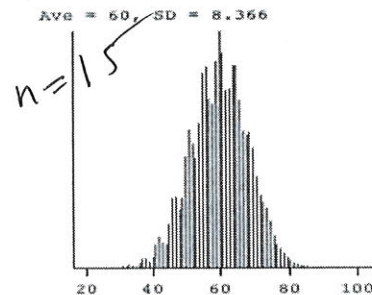
34) Now suppose you're just interested in how many correct answers the student would get by guessing, not his score. Then the **EV = 50 and the SE = 5**. Suppose the student needs to get **55** answers correct in order to pass.

What's the probability the student will pass? (Hint: convert to a Z score, and use the normal curve).

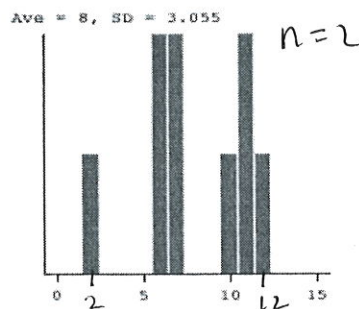
- a) 2.5% b) 5% (c) 6% d) 32% e) 40%

$$Z = \frac{55 - 50}{5} = 1$$

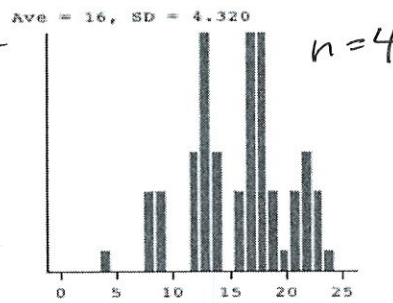
The next 3 questions pertain to the 3 probability histograms below. They depict (in scrambled order) the sums of 2, 4, and 15 draws from the same box.



Histogram 1



Histogram 2



Histogram 3

35) Histogram 2 is the probability histogram for how many draws?

- (a) 2 b) 4 c) 15

36) Histogram 3 is the probability histogram for how many draws?

- a) 2 (b) 4 c) 15

37) The 3 histograms represent the sum of 2, 4 and 15 draws from the **same** box. Which of these boxes is it?

Box A

- a) $\boxed{1} \boxed{5}$

Box B

- (b) $\boxed{1} \boxed{5} \boxed{6}$

Box C

- c) $\boxed{0} \boxed{1} \boxed{5}$

- 38) City A has 4,000,000 people and City B has 40,000 people. A simple random sample of 400 people is taken from City A. In order to keep the accuracy the same, the sample size from City B should be _____ the sample size from City A.
- ☒ a) the same as b) 10 times more than c) 100 times more than d) 10 times less than e) 100 times less than

The next 6 questions pertain to the following poll:

A recent poll asked a sample of 400, randomly chosen from all US adults: "Do you think building a wall between the US and Mexico is necessary to protect the border?" 41% answered "Yes" and 59% answered "No"

- 39) What most closely resembles the relevant box model?
- a) It has 400 tickets, 41% are marked "1" and 59% are marked "0"
- b) It has 400 tickets with an average of 0.
- ☒ c) It has millions of tickets marked "0" and "1", the exact percentage of each is unknown and estimated from the sample.
- 40) Which one of the statements below is true?
- a) The expected value for the percent of registered Democrats who would answer "Yes" to the question is 41%.
- b) The expected value for the percent of immigrants who would answer "Yes" to the question is 41%.
- c) The expected value for the percent of Mexicans who would answer "Yes" to the question is 41%.
- d) All of the above are true.
- ☒ e) None of the above are true.
- 41) Is it possible to compute a 95% confidence interval for the percent of all US adults who would answer "Yes" to the question?
- a) Yes, a 95% confidence interval is approximately $41\% \pm 2.46\%$
- ☒ b) Yes, a 95% confidence interval is approximately $41\% \pm 4.92\%$
- c) No, because we're not given the SD of the sample.
- d) No, because we cannot infer with 95% confidence the answers of 200 million Americans from data based on a sample of only 1,600 randomly selected Americans.
- 42) If the researcher increased his sample size by a factor of 4 (to $n=1600$) then the width of the 95% confidence interval would ...
- a) increase by a factor of 2 b) increase by a factor of 4 ☒ c) decrease by a factor of 2 d) decrease by a factor of 4
- 43) Suppose our sample size was very small (< 10) and we asked the same question and got similar results, would it be appropriate to use Z or t curves to compute Confidence Intervals? a) Z b) t ☒ c) neither
- pop not normal*
never use t for 0-1 box
0-1 box.

The next 3 questions pertain to this situation: To estimate the average Math SAT of students at a large public high school of 2000 students, a random sample of 17 students is taken. The sample average = 500 with a SD = 100. Compute a 95% CI for average SAT score of all 2000 students using the **t distribution**.

- 44) $SE_{avg}^+ =$ a) $\frac{100}{\sqrt{1999}}$ b) $\sqrt{\frac{17}{16}} \times 100$ c) $\frac{100}{\sqrt{17}}$ ☒ d) $\frac{100}{\sqrt{16}} = \frac{SD}{\sqrt{n-1}}$
- 45) A 95% CI = $500 \pm \underline{\hspace{1cm}} \times (SE^+)$ Fill in the blank with the correct number a) 1.96 ☒ b) 2.12 c) 1.75
- t₁₆ for 95% CI is 2.12*
- 46) A 95% CI using the normal distribution would be _____ a 95% CI using the t distribution for the same sample.
- a) wider than ☒ b) narrower than c) the same as

All the question on this page refer to this survey: Suppose a random sample of 100 male and 100 female students chosen from all U of I undergrads were asked: "Do you make your bed every day?" **50%** of the women but only **42%** of the men answered "Yes". Is this 8% difference in the sample large enough to reflect a real difference in the population or is it just due to chance?

First do a 2 sample Z test.

47) The null hypothesis is that the difference between male and female responses in the ...

- a) ☒ population = 0 b) sample = 0 c) sample \neq 0 d) population \neq 0 e) population and in the sample are the same.

48) The SE for the female sample percent = 5% and the SE for the male sample percent is about 5% too. The SE for the difference of the 2 sample percents is closest to ...

- a) 5% b) 10% c) 2.5 % d) 3.16 % e) ☒ 7.07%

$$\sqrt{5^2 + 5^2} = 7.07$$

49) Suppose the Z test-statistic = 1.13, what do you conclude?

- a) ☒ Cannot reject the null. It's plausible that there is no male/female difference on this question among all U of I undergrads.
b) Reject the null and conclude that there is strong evidence that the sample difference reflects a real male/female difference on this question among all U of I undergrads.

Now perform a χ^2 Independence test to test the null that Gender and Bed making are independent in the population. Use the table below to help you answer the questions, but you'll only be graded on your Scantron answers.

| Observed | | | | Expected | | | |
|----------|-----|-----|-------|----------|-----|-----|-------|
| | Yes | No | Total | | Yes | No | Total |
| Male | 42 | 58 | 100 | Male | 46 | 54 | 100 |
| Female | 50 | 50 | 100 | Female | 46 | 54 | 100 |
| Total | 92 | 108 | 200 | Total | 92 | 108 | 200 |

50) The expected counts are computed assuming a) ☒ H_0 is true b) H_0 is false

51) The expected number of males who would answer "No" is

- a) 42 b) 46 c) 50 d) ☒ 54 e) 58

52) How many degrees of freedom for the χ^2 test? a) 1 b) 2 c) 3 d) 4 e) 5 $(2-1)(2-1) = 1$

53) Suppose we got a χ^2 test statistic = 1.28. The p-value is

- a) $< 0.1\%$ b) between 0.1% and 1% c) between 1% and 5% d) between 5% and 10% e) ☒ between 10% and 30%

54) In general the χ^2 Independence test and the _____ will yield about the same p-values.

- a) 1-tailed 2 sample Z test b) ☒ 2-tailed 2 sample Z test

The next 10 questions pertain to this situation: Suppose a city of with a population of 100,000 adults is 40% white, 25% black, 20% Asian and 15% Hispanic. 100 adults were chosen to be on a jury panel. Of the 100, 50 were white, 25 were black, 10 were Asian and 15 were Hispanic. The city claimed the 100 adults were randomly chosen? Does the data support that claim. Do a significance test. Use the table to help you answer the questions but you'll only be graded on your Scantron answers.

4 categories

| | Observed | Expected | Obs - Exp | (Observed - Exp) ² | (Observed - Exp) ² /Exp |
|----------|----------|----------|-----------|-------------------------------|------------------------------------|
| White | 50 | 40 | 10 | 10 ² = 100 | 100/40 = 2.5 |
| Black | 25 | 25 | 0 | 0 | 0 |
| Asian | 10 | 20 | -10 | (-10) ² = 100 | 100/20 = 5 |
| Hispanic | 15 | 15 | 0 | 0 | 0 |
| Sum | 100 | 100 | 0 ✓ | ✓ | 7.5 |

55) Which significance test is appropriate for this situation?

- (a) Only a χ^2 goodness-of-fit test b) Only a χ^2 independence test c) Both χ^2 tests

56) The test statistic is computed to be $\chi^2 = 2.5 + 0 + 5 + 0$. The White and Hispanic terms are missing.

What is the Asian term? a) 0 b) 1.25 (c) 2.5 d) 5 e) Not enough info

57) How many degrees of freedom? a) 1 b) 2 (c) 3 d) 4 e) 5 # categories - 1 = 4 - 1 = 3

58) Look at the χ^2 table: To reject the null at 5%, the χ^2 test statistic must be > 7.81.

- a) 5 b) 5.99 (c) 7.81 c) 9.49 d) 11.07

Now suppose we're only interested in whether Asians are being discriminated against. The city is 20% Asian but only 10% were chosen. Is this strong enough evidence to conclude that the jury wasn't randomly selected? Do a significance test.

59) This translates into drawing at random ____ times a) 10 (b) 100 c) 100,000

60) ____ replacement (a) without b) with

61) from a null box that contains

- (a) 100,000 tickets 20% marked "1" and 80% marked "0"
 b) 100,000 tickets marked either "1" or "0", but the exact percentages of each are unknown and estimated from our sample.
 c) 100 tickets 10 marked "1" and 90 marked "0"
 d) 100,000 tickets 10% marked "1" and 90% marked "0"
 e) 100,000 tickets marked either "40%", "25%", "20%", or "15%"

62) What is the SE_%?

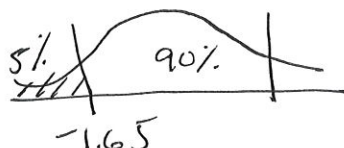
- a) $\frac{\sqrt{(0.15)(0.85)}}{\sqrt{100}} \times 100$ b) $\frac{\sqrt{(0.15)(0.85)}}{\sqrt{100,000}} \times 100$ (c) $\frac{\sqrt{(0.2)(0.8)}}{\sqrt{100}} \times 100$ d) $\frac{\sqrt{(0.2)(0.8)}}{\sqrt{100,000}} \times 100$

63) What is the Z statistic?

- a) $\frac{-20\%}{SE_{\%}}$ b) $\frac{-15\%}{SE_{\%}}$ (c) $\frac{-10\%}{SE_{\%}}$ d) $\frac{-5\%}{SE_{\%}}$

64) To reject the null at significance level 5%, we'd need a Z statistic < ____ for a one-sided test.

- a) -2 (b) -1.65 c) 1.65 d) 2 e) 0

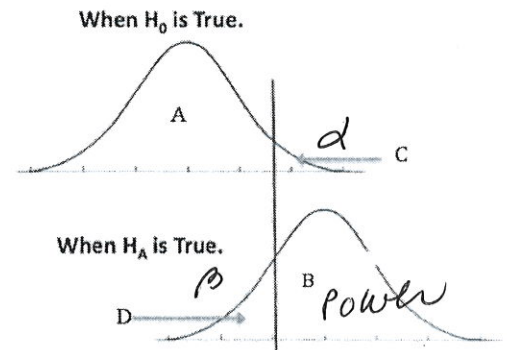


- 65) Suppose a well-designed randomized controlled double-blind experiment is done to test a new drug. The null hypothesis is that the drug works no better than a placebo. A significance test is done and the p-value is computed to be 5%. Which statement **best** describes what is meant by a P value of 5%?
- ☒ It means that even if the null hypothesis was true and the drug didn't work, we would still see evidence this strong or stronger 5% of the time just by chance.
 - ☐ It means there is a 5% chance that the null is true.
 - ☐ It means that there is a 5% chance that the null is false.
 - ☐ It means that we have proof the drug certainly works
- 66) An experiment on ESP is repeated **2000** times. Suppose there is no ESP, and the experiment is done correctly with no cheating. About how many of the experiments would you expect to find statistically significant evidence for ESP, that is how many of the results would get p-values $< 5\%$? (Note, answer *how many*, **not** what percent.)
- a) 0 b) 0.1 c) 5 d) 50 ☒ 100 $0.05 \times 2000 = 100$
- 67) The convention is to reject the null when $p < 5\%$ and call the result "statistically significant". Is there any particular mathematical justification for this?
- ☐ Yes, the shape of the normal curve, the t-curves and the chi-square curves all have sharp dropping off points that make 5% a natural dividing line.
 - ☐ Yes, 5% is the most likely percent to avoid the mistake of rejecting the null when the null is really true. All other percents would yield a higher likelihood of making that mistake.
 - ☒ No, there's no particular mathematical justification for choosing 5%.

The next 4 questions pertain the histograms below and Type I and Type II errors

The histograms below show the sampling distribution of a test statistic under the Null and Alternative Hypotheses of a significance test. Match the labeled areas to their correct descriptions. The vertical line is the null cut-off.

- 68) Type I errors (α) correspond to Area
- a) A b) B ☒ C d) D
- 69) Type II errors (β) correspond to Area
- a) A b) B c) C ☒ D
- 70) Power corresponds to Area
- a) A ☒ B c) C d) D
- 71) If we decrease the probability of a Type I error what happens to the probability of Type II error?
- a) Decreases ☒ b) Increases c) Stays the Same

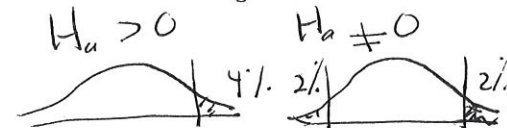


The next 2 questions pertain to:

A large randomized experiment is done to test the effectiveness of some drug. The researchers do a significance test and set the p-value to reject the null at 4%. The null and alternative hypotheses are:

H_0 : The difference in cure rates between the drug and the placebo = 0

H_A : The difference in cure rates between the drug and the placebo $\neq 0$

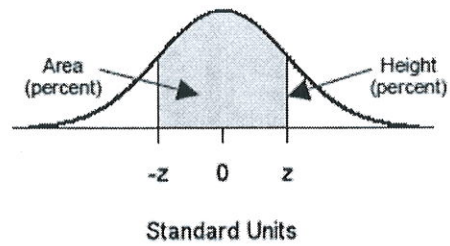


- 72) When the null is true, what's the chance the researchers will make the wrong decision and decide the drug works when it really doesn't
- a) 2% ☒ b) 4% c) 96% d) 98% e) not enough info

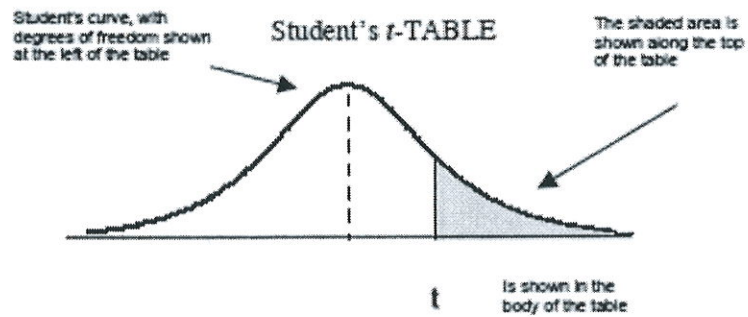
if $\neq 0$ then 2%

- 73) When the null is false and the drug really *does* work what's the chance the researchers will correctly reject the null?
- a) 2% b) 4% c) 96% d) 98% ☒ e) not enough info given
- can't get Type II error from Type I, need more info

STANDARD NORMAL TABLE



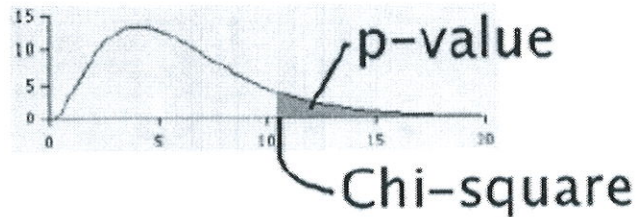
| z | <i>Area</i> | | z | <i>Area</i> | | z | <i>Area</i> |
|------|-------------|--|------|-------------|--|------|-------------|
| 0.00 | 0.00 | | 1.50 | 86.64 | | 3.00 | 99.730 |
| 0.05 | 3.99 | | 1.55 | 87.89 | | 3.05 | 99.771 |
| 0.10 | 7.97 | | 1.60 | 89.04 | | 3.10 | 99.806 |
| 0.15 | 11.92 | | 1.65 | 90.11 | | 3.15 | 99.837 |
| 0.20 | 15.85 | | 1.70 | 91.09 | | 3.20 | 99.863 |
| | | | | | | | |
| 0.25 | 19.74 | | 1.75 | 91.99 | | 3.25 | 99.885 |
| 0.30 | 23.58 | | 1.80 | 92.81 | | 3.30 | 99.903 |
| 0.35 | 27.37 | | 1.85 | 93.57 | | 3.35 | 99.919 |
| 0.40 | 31.08 | | 1.90 | 94.26 | | 3.40 | 99.933 |
| 0.45 | 34.73 | | 1.95 | 94.88 | | 3.45 | 99.944 |
| | | | | | | | |
| 0.50 | 38.29 | | 2.00 | 95.45 | | 3.50 | 99.953 |
| 0.55 | 41.77 | | 2.05 | 95.96 | | 3.55 | 99.961 |
| 0.60 | 45.15 | | 2.10 | 96.43 | | 3.60 | 99.968 |
| 0.65 | 48.43 | | 2.15 | 96.84 | | 3.65 | 99.974 |
| 0.70 | 51.61 | | 2.20 | 97.22 | | 3.70 | 99.978 |
| | | | | | | | |
| 0.75 | 54.67 | | 2.25 | 97.56 | | 3.75 | 99.982 |
| 0.80 | 57.63 | | 2.30 | 97.86 | | 3.80 | 99.986 |
| 0.85 | 60.47 | | 2.35 | 98.12 | | 3.85 | 99.988 |
| 0.90 | 63.19 | | 2.40 | 98.36 | | 3.90 | 99.990 |
| 0.95 | 65.79 | | 2.45 | 98.57 | | 3.95 | 99.992 |
| | | | | | | | |
| 1.00 | 68.27 | | 2.50 | 98.76 | | 4.00 | 99.9937 |
| 1.05 | 70.63 | | 2.55 | 98.92 | | 4.05 | 99.9949 |
| 1.10 | 72.87 | | 2.60 | 99.07 | | 4.10 | 99.9959 |
| 1.15 | 74.99 | | 2.65 | 99.20 | | 4.15 | 99.9967 |
| 1.20 | 76.99 | | 2.70 | 99.31 | | 4.20 | 99.9973 |
| | | | | | | | |
| 1.25 | 78.87 | | 2.75 | 99.40 | | 4.25 | 99.9979 |
| 1.30 | 80.64 | | 2.80 | 99.49 | | 4.30 | 99.9983 |
| 1.35 | 82.30 | | 2.85 | 99.56 | | 4.35 | 99.9986 |
| 1.40 | 83.85 | | 2.90 | 99.63 | | 4.40 | 99.9989 |
| 1.45 | 85.29 | | 2.95 | 99.68 | | 4.45 | 99.9991 |



| Degrees of freedom | 25% | 10% | 5% | 2.5% | 1% | 0.5% |
|--------------------|------|------|------|-------|-------|-------|
| 1 | 1.00 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 |
| 2 | 0.82 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 |
| 3 | 0.76 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 |
| 4 | 0.74 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 |
| 5 | 0.73 | 1.48 | 2.02 | 2.57 | 3.36 | 4.03 |
| 6 | 0.72 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 |
| 7 | 0.71 | 1.41 | 1.89 | 2.36 | 3.00 | 3.50 |
| 8 | 0.71 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 |
| 9 | 0.70 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 |
| 10 | 0.70 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 |
| 11 | 0.70 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 |
| 12 | 0.70 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 |
| 13 | 0.69 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 |
| 14 | 0.69 | 1.35 | 1.76 | 2.14 | 2.62 | 2.98 |
| 15 | 0.69 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 |
| 16 | 0.69 | 1.34 | 1.75 | 2.12 | 2.58 | 2.92 |
| 17 | 0.69 | 1.33 | 1.74 | 2.11 | 2.57 | 2.90 |
| 18 | 0.69 | 1.33 | 1.73 | 2.10 | 2.55 | 2.88 |
| 19 | 0.69 | 1.33 | 1.73 | 2.09 | 2.54 | 2.86 |
| 20 | 0.69 | 1.33 | 1.72 | 2.09 | 2.53 | 2.85 |
| 21 | 0.69 | 1.32 | 1.72 | 2.08 | 2.52 | 2.83 |
| 22 | 0.69 | 1.32 | 1.72 | 2.07 | 2.51 | 2.82 |
| 23 | 0.69 | 1.32 | 1.71 | 2.07 | 2.50 | 2.81 |
| 24 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.80 |
| 25 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.79 |

$$\chi^2 = \sum (\text{obs-exp})^2 / \text{exp}$$

Chi-Square Table



| Degrees of freedom ↓ | 30% | 10% | 5% | 1% | 0.1% | ← p-value |
|----------------------|-------|-------|-------|-------|-------|--------------|
| 1 | 1.07 | 2.71 | 3.84 | 6.63 | 10.83 | ← Chi-square |
| 2 | 2.41 | 4.61 | 5.99 | 9.21 | 13.82 | |
| 3 | 3.66 | 6.25 | 7.81 | 11.34 | 16.27 | |
| 4 | 4.88 | 7.78 | 9.49 | 13.28 | 18.47 | |
| 5 | 6.06 | 9.24 | 11.07 | 15.09 | 20.52 | |
| 6 | 7.23 | 10.64 | 12.59 | 16.81 | 22.46 | |
| 7 | 8.38 | 12.02 | 14.07 | 18.48 | 24.32 | |
| 8 | 9.52 | 13.36 | 15.51 | 20.09 | 26.12 | |
| 9 | 10.66 | 14.68 | 16.92 | 21.67 | 27.88 | |
| 10 | 11.78 | 15.99 | 18.31 | 23.21 | 29.59 | |
| 11 | 12.90 | 17.28 | 19.68 | 24.72 | 31.26 | |
| 12 | 14.01 | 18.55 | 21.03 | 26.22 | 32.91 | |
| 13 | 15.12 | 19.81 | 22.36 | 27.69 | 34.53 | |
| 14 | 16.22 | 21.06 | 23.68 | 29.14 | 36.12 | |
| 15 | 17.32 | 22.31 | 25.00 | 30.58 | 37.70 | |
| 16 | 18.42 | 23.54 | 26.30 | 32.00 | 39.25 | |
| 17 | 19.51 | 24.77 | 27.59 | 33.41 | 40.79 | |
| 18 | 20.60 | 25.99 | 28.87 | 34.81 | 42.31 | |
| 19 | 21.69 | 27.20 | 30.14 | 36.19 | 43.82 | |
| 20 | 22.77 | 28.41 | 31.41 | 37.57 | 45.31 | |
| 21 | 23.86 | 29.62 | 32.67 | 38.93 | 46.80 | |
| 22 | 24.94 | 30.81 | 33.92 | 40.29 | 48.27 | |
| 23 | 26.02 | 32.01 | 35.17 | 41.64 | 49.73 | |
| 24 | 27.10 | 33.20 | 36.42 | 42.98 | 51.18 | |